

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Question Set 1

1. A circle with centre C has equation $x^2 + y^2 + 8x - 2y - 7 = 0$.

Find

- (a) the coordinates of C ,

[2]

$$x^2 + y^2 + 8x - 2y - 7 = 0$$

$$(x^2 + 8x) + (y^2 - 2y) - 7 = 0$$

$$(x^2 + 8x + 16 - 16) + (y^2 - 2y + 1 - 1) - 7 = 0$$

$$(x + 4)^2 + (y - 1)^2 - 16 - 1 - 7 = 0$$

$$(x + 4)^2 + (y - 1)^2 = 24 = (2\sqrt{6})^2$$

$$C(-4, 1)$$

- (b) the radius of the circle.

[1]

$$2\sqrt{6}$$

2. Solve the equation $|2x - 1| = |x + 3|$.

[3]

$$|2x - 1| = |x + 3|$$

$$2x - 1 = x + 3$$

$$x = 4$$

$$2x - 1 = -(x + 3)$$

$$2x - 1 = -x - 3$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

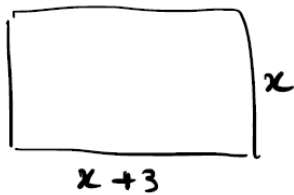
3 In this question you must show detailed reasoning.

A gardener is planning the design for a rectangular flower bed. The requirements are:

- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than 180 m^2 .

The width of the flower bed is x m.

By writing down and solving appropriate inequalities in x , determine the set of possible values for the width of the flower bed. [6]



$$x + 3 \geq 14.5$$

$$x \geq 11.5$$

$$(x + 3)x < 180$$

$$x^2 + 3x - 180 < 0$$

$$! \quad x \quad \begin{matrix} 15 = 15 \\ -12 = -12 \end{matrix}$$

$$(x + 15)(x - 12) < 0$$

$$-15 < x < 12$$

$$11.5 \leq x < 12$$

4

In this question you must show detailed reasoning.

The functions f and g are defined for all real values of x by

$$f(x) = x^3 \quad \text{and} \quad g(x) = x^2 + 2.$$

(a) Write down expressions for

(i) $fg(x)$.

[1]

$$f(x) = x^3 \quad g(x) = x^2 + 2$$

$$f(g(x)) = (x^2 + 2)^3$$

$$= 8 + 12x^2 + 6x^4 + x^6$$

(ii) $gf(x)$.

[1]

$$g(f(x)) = (x^3)^2 + 2 = x^6 + 2$$

(b) Hence find the values of x for which $fg(x) - gf(x) = 24$.

[6]

$$f(g(x)) - g(f(x)) = (8 + 12x^2 + 6x^4 + x^6) - (x^6 + 2)$$
$$= 8 + 12x^2 + 6x^4 + x^6 - x^6 - 2$$

$$24 = 6 + 12x^2 + 6x^4$$

$$6x^4 + 12x^2 - 18 = 0$$

$$x^2 = -3 \text{ or } 1$$

$$6(x^4 + 2x^2 - 3) = 0$$

~~X~~

$$6(x^2 + 3)(x^2 - 1) = 0$$

$$x = \pm 1$$

5

(a) Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2+\sqrt{x}} dx \approx \frac{11}{4} - \sqrt{2}.$$

[5]

$$x = 0 \sim 2 \Rightarrow 2$$

$$y = \frac{1}{2}, y = \frac{1}{2+\sqrt{2}}$$

$$x = 2 \sim 4 \Rightarrow 2$$

$$y = \frac{1}{2+\sqrt{2}}, y = \frac{1}{4}$$

$$\int_0^4 \frac{1}{2+\sqrt{x}} dx = \frac{1}{2} \times 2 \times \left(\frac{1}{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{2+\sqrt{2}} + \frac{1}{4} \right)$$

$$= \frac{3}{4} + \frac{2}{2+\sqrt{2}} = \frac{3}{4} + \frac{2(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$$

$$= \frac{3}{4} + \frac{4-2\sqrt{2}}{4-2} = \frac{3}{4} + 2 - \sqrt{2}$$

$$= \frac{11}{4} - \sqrt{2}$$

(b) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{x}} dx. \quad [6]$$

$$\begin{aligned} & \int_0^4 \frac{1}{2+\sqrt{x}} dx & x &= u^2 & \frac{du}{dx} &= \frac{1}{2\sqrt{x}} & dx &= 2\sqrt{x} du \\ & & u &= \sqrt{x} & & & &= 2u du \\ & = \int_0^2 \frac{1}{2+u} \times 2u du & u &= \sqrt{4} = 2 & u &= \sqrt{0} = 0 & & \\ & = \int_0^2 \frac{2u}{2+u} du = 2 \int_0^2 \frac{u}{2+u} du & & & v &= 2+u & & \\ & = 2 \int_2^4 \frac{v-2}{v} dv = 2 \int_2^4 1 dv - 4 \int_2^4 \frac{1}{v} dv & & & \frac{dv}{du} &= 1 & du &= dv \\ & = 2[v]_2^4 - 4[\ln v]_2^4 & & & v &= 2+2=4 & v &= 2+0=2 \\ & = 2(4-2) - 4(\ln 4 - \ln 2) \\ & = 4 - 4 \ln\left(\frac{2^2}{2}\right) = 4 - 4 \ln 2 = 2(2 - 2 \ln 2) \end{aligned}$$

(c) Using your answers to parts (a) and (b), show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where k is a rational number to be determined. [2]

$$\begin{aligned} \frac{11}{4} - \sqrt{2} &= 4 - 4 \ln 2 \\ 4 \ln 2 &= \sqrt{2} + \frac{5}{4} \\ \ln 2 &= \frac{5}{16} + \frac{\sqrt{2}}{4} \end{aligned}$$

6 It is given that the angle θ satisfies the equation $\sin\left(2\theta + \frac{1}{4}\pi\right) = 3 \cos\left(2\theta + \frac{1}{4}\pi\right)$.

(a) Show that $\tan 2\theta = \frac{1}{2}$. [3]

$$\begin{aligned} \sin\left(2\theta + \frac{1}{4}\pi\right) &= 3 \cos\left(2\theta + \frac{1}{4}\pi\right) \\ \sin 2\theta \cos \frac{1}{4}\pi + \cos 2\theta \sin \frac{1}{4}\pi &= 3 \cos 2\theta \cos \frac{1}{4}\pi - 3 \sin 2\theta \sin \frac{1}{4}\pi \\ \sin 2\theta \cos \frac{1}{4}\pi + 3 \sin 2\theta \sin \frac{1}{4}\pi &= -\cos 2\theta \sin \frac{1}{4}\pi + 3 \cos 2\theta \cos \frac{1}{4}\pi \\ \sin 2\theta (\cos \frac{1}{4}\pi + 3 \sin \frac{1}{4}\pi) &= \cos 2\theta (3 \cos \frac{1}{4}\pi - \sin \frac{1}{4}\pi) \\ \frac{\sin 2\theta}{\cos 2\theta} &= \frac{3 \cos \frac{1}{4}\pi - \sin \frac{1}{4}\pi}{\cos \frac{1}{4}\pi + 3 \sin \frac{1}{4}\pi} = \frac{1}{2} \\ \tan 2\theta &= \frac{1}{2} \end{aligned}$$

(b) Hence find, in surd form, the exact value of $\tan \theta$, given that θ is an obtuse angle.

[5]

$$\tan 2\theta = \frac{\tan \theta + \tan \theta}{1 - \tan \theta \tan \theta} = \frac{1}{2}$$

$$2 \tan \theta = 2(1 - \tan^2 \theta)$$

$$\tan^2 \theta + 2 \tan \theta - 2 = 0$$

$$\tan \theta = \frac{-2 \pm \sqrt{-2 - 4(1)(-2)}}{2 \times 1} = \frac{-2 \pm \sqrt{6}}{2}$$

$$\tan \theta = \frac{-2 - \sqrt{6}}{2}$$

7 The gradient of the curve $y = f(x)$ is given by the differential equation

$$(2x-1)^3 \frac{dy}{dx} + 4y^2 = 0$$

and the curve passes through the point $(1, 1)$. By solving this differential equation show that

$$f(x) = \frac{ax^2 - bx + 1}{bx^2 - bx + 1},$$

where a and b are integers to be determined.

[9]

$$(2x-1)^3 \frac{dy}{dx} + 4y^2 = 0$$

$$(2x-1)^3 dy + 4y^2 dx = 0$$

$$-\frac{1}{(2x-1)^3} dx = \frac{1}{4y^2} dy$$

$$-\int \frac{1}{(2x-1)^3} dx = \frac{1}{4} \int \frac{1}{y^2} dy$$

$$-\int \frac{1}{u^3} \times \frac{du}{2} = \frac{1}{4} \int y^{-2} dy$$

$$-\frac{1}{2} \times -\frac{1}{2} u^{-2} = -\frac{1}{4} y^{-1} + c$$

$$\frac{1}{4u^2} = -\frac{1}{4y} + c$$

$$\frac{1}{4(2x-1)^2} = -\frac{1}{4y} + c$$

$$u = 2x-1 \quad \frac{du}{dx} = 2 \quad dx = \frac{du}{2}$$

$$(1, 1) \quad \frac{1}{4(2(1)-1)^2} = -\frac{1}{4(1)} + c$$

$$c = \frac{1}{2}$$

$$\frac{1}{4(2x-1)^2} = -\frac{1}{4y} + \frac{1}{2}$$

$$8y = -8(2x-1)^2 + 16y(2x-1)^2$$

$$(16(2x-1)^2 - 8)y = 8(2x-1)^2$$

$$y = \frac{8(4x^2 - 4x + 1)}{16(4x^2 - 4x + 1) - 8}$$

$$f(x) = \frac{4x^2 - 4x + 1}{8x^2 - 8x + 1}$$

Total Marks for Question Set 1: 50 Marks

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